1. A special fully discrete 2-year endowment insurance with a maturity value of 1000 is issued to (x). The death benefit in each year is 1000 plus the benefit reserve at the end of that year.

   You are given:
   i) $i = .05$ ii) $q_{x+k} = .10(1.10)^k$, k = 0, 1

5 Calculate the net level annual premium. 564.46

2. A fully discrete \$1,000 whole life policy is issued to (40). The mean and standard deviation of the prospective loss variable after 2 years are \$100 and \$546.31, respectively. The mean and standard deviation of the prospective loss variable after 11 years are \$300 and \$300, respectively.

   You are given:
   i) $q_{41} = .40$
   ii) $q_{41+t} = q_{41}\,(1.01)^t$
   iii) $d = 6\%$

2 X .09344213

Find $\dfrac{Var({}_{10}L \mid K \geq 10)}{Var({}_{1}L \mid K \geq 1)}$.

3. Assuming a uniform distribution of deaths in each year of age, you are given:
   i) ${}_hV^{(2)} = 0.3$
   ii) ${}_{h+1}V^{(2)} = 0.5$
   iii) $\pi_h^{(2)} = 0.4$
   iv) $v = 0.95$
   v) $p_{x+h} = 0.90$

4 X .6688989

Determine the benefit reserve at the end of the first month of the year, ${}_{h+1/12}V^{(2)}$

4. A fully discrete \$100,000 endowment-to-age-100 policy is issued to (40). The death benefit is defined as follows:

$$b_{h+1} = {}_{h+1}V + \frac{1000}{q_{40+h}}$$

0 X

Find the level benefit premium if $i = 10\%$.

1530.30303 714.06707

~~2120.8024~~

1005.47

5. Let L denote the insurer's loss for a fully discrete annual premium two-year endowment of \$10 issued to (x). Premiums are determined by the equivalence principle, but are not necessarily level.

3

You are given:
i) $d = 10\%$
ii) $Var(L) = 3.24$
iii) ${}_1V = 5.00$ 5,188559
iv) $p_x > q_x$

X

Calculate the first year premium.

6. ${}_kL$ is the prospective-loss-at-time-k random variable for a fully discrete, level premium, 3-year endowment insurance of 15 issued to (x).

5

You are given:
i) $i = 20\%$
ii) The reserve at the end of the first year is 3.30.
iii) The reserve at the end of the second year is 7.80. 6.75

Calculate $\text{Var}({}_1L \mid K(x) \geq 1)$.

7. For a fully continuous whole life insurance issued to (40), you are given:

i) The death benefit of 100,000 is payable at the moment of death.
ii) Benefit premiums are payable continuously at t at the annual rate $\pi_t = \pi e^{0.05t}$.
iii) Mortality follows De Moivre's law with $\mu(x) = 1/(100 - x)$
iv) $\delta = 0.05$

2

X Calculate π. 527.896

8. For a fully discrete 5-year-deferred life annuity-due on (25), level premiums are payable at the beginning of each year for the next 5 years. Assume aggregate mortality (i.e. there is no select period).

3

You are given:

i) $\ddot{a}_{25} = 12$

ii) $v = 0.93$ P=1.8621656

X iii)

k	1	2	3	4	5
${}_kp_x$	.99	.98	.97	.96	.95

9.81173

Calculate the reserve at the end of the 4th policy year, denoted ${}_4V$.

9. For a fully discrete 10-payment whole life insurance of 1000 on (x), you are given:

i) $i = .06$

ii) $q_{x+9} = .01262$

5 iii) The annual benefit premium is 32.88

iv) The benefit reserve at the end of year 9 is 322.87

Calculate $1000P_{x+10}$, the benefit premium for a fully discrete whole life insurance of 1000 on $(x+10)$.

33.12

10. A fully discrete 10-year decreasing term insurance is issued to (40) and pays a death benefit of $10,000 in the first year, $9000 in the second year, and so on. The level annual premium is $80.

You are given that $v = .95$, $q_{40} = q_{41}$, and the benefit reserve at the end of the second policy year is 0.

5

Find the benefit reserve at the end of the first policy year.

−4.3304

ACT 3230 Actuarial Models II

Exam 3 Solutions – Chapter 8

March 17, 2008
4:00 p.m. – 5:15 p.m.

Instructor: Sheldon Liu, FSA, FCIA

1.

Note that ${}_0V = 0$ (equivalence principle) and ${}_2V = 1000$ (since the maturity value is payable and no more premiums are expected).

$$({}_hV + \pi_h)(1+i) = b_{h+1}\, q_{x+h} + p_{x+h}\, {}_{h+1}V$$

Year 1:
$$({}_0V + P)(1+i) = (1000 + {}_1V)\, q_x + p_x\, {}_1V$$
$$P(1.05) = .1(1000 + {}_1V) + .9({}_1V)$$
$$1.05P - 100 = {}_1V$$

Year 2:
$$({}_1V + P)(1+i) = (1000 + {}_2V)\, q_{x+1} + p_{x+1}\, {}_2V$$
$$(1.05P - 100 + P)(1.05) = .11(1000 + 1000) + .89(1000)$$
$$2.05P - 100 = 1110 / 1.05$$
$$P = (1110 / 1.05 + 100) / 2.05 = \mathbf{564.46}$$

2.

$$E({}_2L \mid K(40) \geq 2) = 100 = {}_2V_{40} \qquad Var({}_2L \mid K(40) \geq 2) = (546.31)^2$$
$$E({}_{11}L \mid K(40) \geq 11) = 300 = {}_{11}V_{40} \qquad Var({}_{11}L \mid K(40) \geq 11) = (300)^2$$

$$q_{50} = (.40)(1.01)^9 = 0.437474109$$

$$Var({}_hL \mid K(x) \geq h) = Var(\Lambda_h \mid K(x) \geq h) + v^2\, p_{x+h} Var({}_{h+1}L \mid K(x) \geq h+1)$$

$$Var({}_{10}L \mid K(40) \geq 10) = Var(\Lambda_{10} \mid K(40) \geq 10) + v^2\, p_{40+10} Var({}_{11}L \mid K(40) \geq 11)$$
$$= v^2\,(b_{11} - {}_{11}V_{40})^2\, p_{40+10}\, q_{40+10} + v^2\, p_{50}\,(300)^2$$
$$= (1 - .06)^2 (1000 - 300)^2 (1 - .437474)(.437474) + (1 - .06)^2 (1 - .437474)(300)^2$$
$$= 151{,}282.64$$

$$Var({}_1L \mid K(40) \geq 1) = Var(\Lambda_1 \mid K(40) \geq 1) + v^2\, p_{40+1} Var({}_2L \mid K(40) \geq 2)$$
$$= v^2\,(b_2 - {}_2V_{40})^2\, p_{41}\, q_{41} + v^2\, p_{41}\, Var({}_2L \mid K(40) \geq 2)$$
$$= (.94)^2 (1000 - 100)^2 (1 - .40)(.40) + (.94)^2 (1 - .40)(546.31)^2$$
$$= 330{,}000.54$$

$$\frac{Var({}_{10}L \mid K \geq 10)}{Var({}_{1}L \mid K \geq 1)} = \frac{151{,}282.64}{330{,}000.54} = \mathbf{0.458431}$$

3.

Re: Under UDD, ${}_sq_x = s \cdot q_x$

$${}_{1/12}p_{x+h} = 1 - {}_{1/12}q_{x+h} = 1 - (\tfrac{1}{12}) \cdot q_{x+h} = 1 - (1/12)(1 - 0.90) = 0.99166$$

$${}_{0.5}p_{x+h} = 1 - {}_{0.5}q_{x+h} = 1 - (0.5) \cdot q_{x+h} = 1 - (0.5)(1 - 0.90) = 0.95$$

$$v^s\,{}_sp_{x+h}\,{}_{h+s}V^{(2)} = ({}_hV^{(2)} + \frac{\pi_h^{(2)}}{2})(1-s) + [{}_{h+1}V^{(2)}\,v\,p_{x+h} - \frac{\pi_h^{(2)}}{2}(v^{0.5})\,{}_{0.5}p_{x+h}](s)$$

$$(.95)^{1/12}(0.99166)\,{}_{h+1/12}V^{(2)} = (0.3 + \frac{0.4}{2})(1 - 1/12) + [0.5(.95)(.90) -$$

$$\frac{0.4}{2}(.95)^{0.5}(0.95)](1/12)$$

$$(0.987436892)\;{}_{h+1/12}V^{(2)} = (0.5)(11/12) + (0.242310907)(1/12)$$

$${}_{h+1/12}V^{(2)} = 0.478525909 / 0.987436892 = \mathbf{0.484614169}$$

4.

$${}_kV = \sum_{h=0}^{k-1} [\pi_h - (b_{h+1} - {}_{h+1}V)\,vq_{x+h}]\,(1+i)^{k-h}$$

$${}_{60}V = \sum_{h=0}^{k-1} [\pi - ({}_{h+1}V + \frac{1000}{q_{40+h}} - {}_{h+1}V)\,vq_{40+h}]\,(1+i)^{k-h}$$

$$100{,}000 = \sum_{h=0}^{59} \pi(1+i)^{60-h} - \sum_{h=0}^{59} 1000v(1+i)^{60-h}$$

$$100{,}000 = \pi\sum_{h=0}^{59} (1.10)^{60-h} - 1000\sum_{h=0}^{59} (1.10)^{60-h-1}$$

$$100{,}000 = \pi\,\ddot{s}_{\overline{60}|i=.1} - 1000\,s_{\overline{60}|i=.1}$$

$$100{,}000 = \pi\left[\frac{(1.1)^{60}-1}{.1}(1.1)\right] - 1000\left[\frac{(1.1)^{60}-1}{.1}\right]$$

$$100{,}000 = 3{,}338.298\,\pi - 3{,}034{,}816.395$$

$\pi = \mathbf{939.05}$

5.

$$Var({}_hL \mid K(x) \geq h) = Var(\Lambda_h \mid K(x) \geq h) + v^2\, p_{x+h} Var({}_{h+1}L \mid K(x) \geq h+1)$$

Note that $Var({}_1L \mid K(x) \geq 1) = 0$ since 10 will be paid whether the life dies or survives.

$$Var({}_0L \mid K(x) \geq 0) = Var(\Lambda_0 \mid K(x) \geq 0) + v^2\, p_x Var({}_1L \mid K(x) \geq 1)$$
$$3.24 = v^2\,(b_1 - {}_1V)^2\, p_x\, q_x + v^2\, p_x\,(0)$$
$$3.24 = (1 - .1)^2\,(10 - 5)^2\, p_x\, q_x$$
$$0.16 = p_x\, q_x = p_x(1 - p_x)$$
$$\rightarrow p_x = 0.8 \text{ or } 0.2$$

If this is not immediately obvious, you can use the quadratic formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

And since by (iv) $p_x > q_x$, $p_x = 0.8$

$${}_hV + \pi_h = v\, b_{h+1}\, q_{x+h} + v\, p_{x+h}\ {}_{h+1}V$$
$${}_0V + \pi_0 = v\, b_1\, q_x + v\, p_x\ {}_1V$$
$$0 + \pi_0 = (1 - .1)(10)(.2) + (1 - .1)(.8)(5)$$
$$\pi_0 = \mathbf{5.4}$$

Alternatively, you could recognize:

$${}_1V + \pi_1 = v\, b_2\, q_{x+1} + v\, p_{x+1}\ {}_2V$$
$$5 + \pi_1 = (1 - .1)(10)(q_{x+1} + p_{x+1})$$ - note that ${}_2V = b_2 = 10$ and $q_{x+1} + p_{x+1} = 1$
$$\pi_1 = 4$$

And by equivalence principle,
APV of FP = APV of FB
$$\pi_0 + \pi_1\, v\, p_x = 10\, v q_x + 10\, v^2 p_x$$
$$\pi_0 + (4)(1 - .1)(.8) = 10(1 - .1)(.2) + 10(1 - .1)^2(.8)$$
$$\pi_0 = \mathbf{5.4}$$

6.

$${}_hV + \pi_h = v\, b_{h+1}\, q_{x+h} + v\, p_{x+h}\ {}_{h+1}V$$

First, calculate the premium, using the fact that there is no mortality risk in the third year.
$${}_2V + \pi = (1.20)^{-1}\,(15)\, q_{x+2} + (1.20)^{-1}\,(15)\, p_{x+2}$$
$$\pi = 15 / 1.20 - 7.8 = 4.7$$

Then, calculate q_{x+1}.

$${}_1V + \pi = (1.20)^{-1}(15)\, q_{x+1} + (1.20)^{-1}(7.80)\, p_{x+1}$$
$$3.30 + 4.7 = 12.5\, q_{x+1} + 6.5(1 - q_{x+1})$$
$$q_{x+1} = (8.0 - 6.5)/6 = 0.25$$

If death occurs in year 2,

$${}_1L = v\, b_2 - \pi = 15/1.20 - 4.7 = 7.8$$

If death does not occur in year 2 (benefit will be paid at end of year 3 regardless of death/survival in year 3),

$${}_1L = v^2\, b_3 - \pi - v\pi = 15/1.20^2 - 4.7 - 4.7/1.20 = 1.8$$

$$E({}_1L \mid K(x) \geq 1) = (.25)(7.8) + (1 - .25)(1.8) = 3.3$$
$$E({}_1L^2 \mid K(x) \geq 1) = (.25)(7.8)^2 + (1 - .25)(1.8)^2 = 17.64$$

$$Var({}_1L \mid K(x) \geq 1) = E({}_1L^2 \mid K(x) \geq 1) - E({}_1L \mid K(x) \geq 1)^2$$
$$= 17.64 - (3.3)^2 = \mathbf{6.75}$$

Alternatively, recognizing that ${}_1L \mid K(x) \geq 1$ is a Bernoulli random variable,

$$Var({}_1L \mid K(x) \geq 1) = (0.25)(0.75)(7.8 - 1.8)^2 = \mathbf{6.75}$$

7.

Re: Under DML, $\bar{A}_x = \dfrac{\bar{a}_{\overline{\omega-x}|}}{\omega - x}$ and $\overset{\circ}{e}_{40} = (\omega - x)/2$

$$\bar{A}_{40} = \frac{\bar{a}_{\overline{100-40}|}}{100-40} = \frac{(1-e^{-.05(60)})/.05}{60} = 0.316737644$$

APV of FP: $\bar{a}^{\bullet}_{40} = \int_0^\infty \pi_t v^t\, {}_tp_x\, dt = \int_0^\infty \pi e^{0.05t} e^{-0.05t}\, {}_tp_{40}\, dt = \pi \int_0^\infty {}_tp_{40}\, dt = \pi\, \overset{\circ}{e}_{40}$

$$\overset{\circ}{e}_{40} = (\omega - x)/2 = (100 - 40)/2 = 30$$

Equivalence principle: $100{,}000\, \bar{A}_{40} = \bar{a}^{\bullet}_{40} = \pi\, \overset{\circ}{e}_{40}$

$\rightarrow \pi = (100{,}000)(0.316737644)/30 = \mathbf{1055.79}$

8.

$$\ddot{a}_{25:\overline{5}|} = 1 + v\,{}_1p_x + v^2\,{}_2p_x + v^3\,{}_3p_x + v^4\,{}_4p_x$$
$$= 1 + (.93)(.99) + (.93)^2(.98) + (.93)^3(.97) + (.93)^4(.96)$$
$$= 4.26665822$$

$_5E_{25} = v^5\,{}_5p_x = (.93)^5(.95) = 0.660903951$

$_{5|}\ddot{a}_{25} = \ddot{a}_{25} - \ddot{a}_{25:\overline{5|}} = 12 - 4.26665822 = 7.73334178$

$P({}_{5|}\ddot{a}_{25}) = {}_{5|}\ddot{a}_{25} \,/\, \ddot{a}_{25:\overline{5|}} = 7.73334178 / 4.26665822 = 1.812505568$

$\ddot{a}_{30} = {}_{5|}\ddot{a}_{25} \,/\, {}_5E_{25} = 7.73334178 / 0.660903951 = 11.70115835$

$_{k+1}p_x = {}_kp_x\,({}_1p_{x+k})$

$p_{29} = {}_5p_x \,/\, {}_4p_x = .95 / .96$

$_4V + P({}_{5|}\ddot{a}_{25}) = v\,q_{25+4}\,(b_5) + v\,p_{25+4}\,({}_5V)$ // ${}_5V = \ddot{a}_{30}$ since no more premiums

$_4V + 1.812 = (.93)(1 - .95/.96)(0) + (.93)(.95/.96)(\ddot{a}_{30})$

$_4V = (.93)(.95/.96)(11.701) - 1.812 = \mathbf{8.956216722}$

9.

${}^{10}_{10}V_x = A_{x+10}$ since no premium is due after age $x+10$.

$({}_hV + \pi_h)(1+i) = b_{h+1}\,q_{x+h} + p_{x+h}\,{}_{h+1}V$

Work with unit benefit first (can multiply by face amount later, since level):

$({}^{10}_{9}V_x + {}_{10}P_x)(1+i) = (1)\,q_{x+9} + p_{x+9}\,({}^{10}_{10}V_x)$

$(.32287 + .03288)(1.06) = .01262 + (1 - .01262)(A_{x+10})$

$A_{x+10} = 0.369133$

$$1000P_{x+10} = 1000\,(A_{x+10} / \ddot{a}_{x+10}) = (1000)\frac{dA_{x+10}}{1 - A_{x+10}} = (1000)\frac{\frac{.06}{1.06}(.369133)}{1 - .369133} = \mathbf{33.12}$$

10.

$_hV + \pi_h = v\,b_{h+1}\,q_{x+h} + v\,p_{x+h}\,{}_{h+1}V$

$$_1V = \frac{({}_0V + \pi)(1+i) - b_1q_x}{p_x} = \frac{(0+80)(.95)^{-1} - 10{,}000q_x}{p_x}$$

$$_2V = \frac{({}_1V + \pi)(1+i) - b_2q_{x+1}}{p_{x+1}} = \frac{[(80)(.95)^{-1} - 10{,}000q_x + 80](.95)^{-1} - 9{,}000q_x}{p_x} = 0$$

→ $q_x = 0.00885$

$$\therefore\ {}_1V = \frac{(80)(.95)^{-1} - 10{,}000(.00885)}{1 - .00885} = \mathbf{-4.33}$$

Alternative:

$${}_1V + \pi = v\, b_2\, q_{40+1} + v\, p_{40+1}\, {}_2V$$
$${}_1V + 80 = (.95)(9000)q_{41} + (.95)p_{41}(0)$$
$$q_{41} = q_{40} = \frac{{}_1V + 80}{8550}$$

$${}_0V + \pi = v\, b_1\, q_{40} + v\, p_{40}\, {}_1V$$
$$0 + 80 = (.95)(10{,}000)\left(\frac{{}_1V+80}{8550}\right) + (.95)\left(1 - \frac{{}_1V+80}{8550}\right){}_1V$$
$$684{,}000 = 9500({}_1V + 80) + .95(8550 - {}_1V - 80)\,{}_1V$$
$$684{,}000 = 9500\,{}_1V + 760{,}000 + 8046.5\,{}_1V - .95\,{}_1V^2$$
$$.95\,{}_1V^2 - 17{,}546.5\,{}_1V - 76{,}000 = 0$$

$${}_1V = \frac{17{,}546.5 \pm \sqrt{(-17{,}546.5)^2 - 4(.95)(-76{,}000)}}{2(.95)} = \frac{17{,}546.5 \pm 17{,}554.72763}{2(.95)}$$
$$= 18{,}474.33 \quad \text{or} \quad -4.33$$

Check: If ${}_1V = 18{,}474.33$, then $q_{40} = \frac{{}_1V+80}{8550} = \frac{18{,}474.33+80}{8550} = 2.17$ but $0 \le q_{40} \le 1$

If ${}_1V = -4.33$, then $q_{40} = \frac{{}_1V+80}{8550} = \frac{-4.33+80}{8550} = 0.00885$ // more reasonable

Note also that by the retrospective approach, the reserve at time 1 is the accumulation of the 1st premium (80) less the cost of insurance (which is never negative). It is hard to imagine \$80 accumulating to some amount greater than 18,474.33.

$\therefore\ {}_1V = \mathbf{-4.33}$